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$\frac{40}{40}$

MTH 213, Exam One

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$0<x<792$

QUESTION 1. (8 points) Given $0 < x < 65$ such that $x\ (mod\ 8) = 4$, $x\ (mod\ 11) = 8$, and $x\ (mod\ 9) = 7$. Find x. (show the work)

$m = 8 \times 11 \times 9 = 792$

$x(\text{mod} 8) = 4$

$m_1 = 8$

$n_1 = \frac{m}{m_1} = 11 \times 9$

$= 99$

$n_1 y_1 = 1$ in Z_8

$99 y_1 = 1$ in Z_8

$3 y_1 = 1$ in Z_8

$\boxed{y_1 = 3}$ $r_1 = 4$

$x(\text{mod } 11) = 8$

$m_2 = 11$

$n_2 = \frac{m}{m_2}$

$= 72$

$n_2 y_2 = 1$ in Z_{11}

$72 y_2 = 1$ in Z_{11}

$6 y_1 = 1$ in Z_{11}

$\boxed{y_1 = 2}$ $r_2 = 8$

$x(\text{mod } 9) = 7$

$m_3 = 9$

$n_3 = \frac{m}{m_3}$

$= 88$

$n_3 y_3 = 1$ in Z_9

$88 y_3 = 1$ in Z_9

$7 y_3 = 1$ in Z_9

$\boxed{y_3 = 4}$ $r_3 = 7$

$\therefore x = (n_1 y_1 r_1 + n_2 y_2 r_2 + n_3 y_3 r_3) \text{ mod } m$

$= (99 \times 3 \times 4 + 2 \times 8 \times 72 + 4 \times 7 \times 88) \text{ mod } 792$

$= (1188 + 1152 + 2464) \text{ mod } 792$

$= (4804) \text{ mod } 792 = 52$

$\therefore x = 52$

QUESTION 2. Write down T or F CLEARLY, (5 points)

(i) $x^2 - 9 = 0$ for some $x \in N^*$ if and only if $y^2 + 3 = 0$ for some $y \in R$. T F False

(ii) $\exists x \in N^*$ such that $y + x \geq 9$ for every $y \in N^*$. True

(iii) If $x^2 = 4$ for some $x \in Z$, then $xy = 2y$ for every $y \in R$. T False

(iv) $\exists !\, w \in N$ such that $w^2 - 2w - 3 = 0$. True

(v) $\forall x \in Z, \exists! y \in Z$ such that $x + y = 3$. True

QUESTION 3. (6 points) Use the 4th method and prove that $\sqrt{45}$ is irrational.

Deny. Let us say that $\sqrt{45}$ is rational, then $\sqrt{45}$ can be written as $\frac{odd}{odd}$ or ($\frac{a}{b}$ where a, b are odd and $gcd(a,b)=1$)

$\therefore \sqrt{45} = \frac{2n+1}{2m+1}$ where n, m are integers

$45 = \frac{(2n+1)^2}{(2m+1)^2} = \frac{4n^2+4n+1}{4m^2+4m+1}$

$\therefore 45(4m^2+4m+1) = 4n^2+4n+1$

$45(4)m^2 + 45(4)m + 45 - 1 = 4n^2+4n$

$45(4m^2) + 45(4m) + 44 = 4n^2+4n$ dividing by 4 we get

$45m^2 + 45m + 11 = n^2+n$

$\underbrace{\underbrace{45(m^2+m)}_{even} + 11}_{\Rightarrow\ ODD} = \underbrace{n^2+n}_{EVEN}$

$\therefore$ A: even $\neq$ odd $\therefore$ our contradiction is wrong Hence $\sqrt{45}$ is irrational

QUESTION 4. (6 points) Solve over planet Z_{12}, $9x = 6$.

Z_{12}, $9x = 6$ $n = 12$ $a = 9$ $b = 6$ $gcd(a,n) = gcd(9,12) = 3$

$3/6$ ✓ $\therefore$ There are 3 solutions $d = \frac{n}{gcd} = \frac{12}{3} = 4$ (interval)

$\therefore S = \{2, 6, 10\}$ are the solutions

QUESTION 5. (6 points)

For $k = 5$ *to* n^2+4 *do*

$x = k^6 + 2 * k^3 + 7$ → $k*k*k*k*k*k + 2*k*k*k + 7 = 10$

For $i = 1$ *to* $3k+2$ *do*

$y = i^7 + 7 * i + k^5$ $i*i*i*i*i*i*i + 7*i + k*k*k*k*k$ $= 13$

Next i

Next k

a) Find the exact number of arithmetic operations that are executed by the code.

Outer Loop	Inner loop	
No. of terms : $n^2+4-5+1 = n^2$	No. of terms $= 3k+2-1+1 = 3k+2$	$\therefore$ Total arithematic $=$
No. of arithematic ops $= 10$	No. of arithematic ops $= 13$	$10(n^2) + \left(\frac{17(13) + (3n^2+14)(13)}{2}\right) \times 1$
$\therefore$ Total operations $= 10(n^2)$	First term : $k = 5$ $\therefore 17(13)$	
	Last term : $3k+2 : k = n^2+4$	
	$3(n^2+4)+2 = (3n^2+14)(13)$	

(b) Find $O(code)$ (i.e., complexity of the code)

$O(code) = n^4$

QUESTION 6. (6 points) Use Math Induction and show that $8 \mid (3^{2n}-1)$ for every $n \geq 1$.

Step 1: Prove for $n=1$: $3^2-1 = 8/1$ $\therefore$ True

Step 2: Let us assume $(3^{2n}-1) = 8k$ for some $n \geq 1$. and $k \in \mathbb{Z}$

Step 3: Prove for $n+1$:

$$3^{2(n+1)}-1 = 3^{2n+2}-1 = 3^{2n}\cdot 3^2 - 1 + 3^2 - 3^2$$

$$= 3^{2n}\cdot 3^2 - 3^2 + 3^2 - 1$$

$$= 3^2\underbrace{(3^{2n}-1)}_{=8k \text{ by } (2)} + \underbrace{3^2-1}_{=8 \text{ by } (1)}$$

$$= 8k(3^2)+8$$

$$= 8(9k+1) \quad \text{Let } 9k+1 = m,\ m \in \mathbb{Z}$$

$$\therefore 3^{2(n+1)}-1 = 8m\ ;\ n+1 \text{ is true}$$

$\therefore$ We can conclude that $8 \mid (3^{2n}-1)$ for all $n \geq 1$

QUESTION 7. (3 points) Let $D = \{1 \leq a < 264 \mid gcd(a,264) = 4\}$. Find $|D|$. $n = 264 = 2^3 \times 3 \times 11$

$$|D| = \phi(n/4) = \phi(2\times 3\times 11) = (2-1)(2)^{1-1}\times(3-1)3^{1-1}\times(11-1)(11)^{1-1}$$

$$= 1\times 2\times 10 = 20$$

Faculty information

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